2.1 Product and Quotient Rules: Quiz B Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QB.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Quiz B

Item	Answer	Rubric / trap
1(a)	$2x\sin x + (x^2+1)\cos x$	product rule: $(2x)(\sin x) + (x^2+1)(\cos x)$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* multiplying the derivatives gives $2x\cos x$; earns 0.
1(b)	$\dfrac{2x^2-2x}{(2x-1)^2}$	quotient rule: $\dfrac{2x(2x-1)-x^2(2)}{(2x-1)^2}$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* the derivative of $2x-1$ is 2, not 1; reversing the numerator flips the sign.
1(c)	0 (for every x in the domain of f)	$\dfrac{5(2x-3)}{2(2x-3)} = \frac{5}{2}$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant in disguise. The quotient rule is valid but long; its numerator simplifies to 0.
1(d)	$3\sec x\tan x$	$\tan x\cot x = 1$ wherever both are defined, so $f(x) = 3\sec x + 1$. The product rule on $\tan x\cot x$ also gives 0: $\sec^2 x\cot x - \tan x\csc^2 x = \csc x\sec x - \csc x\sec x$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant hidden in a product; sec has derivative $\sec x\tan x$, with a plus sign.
1(e)	0	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ are numbers, so $f(x) = \frac{3}{2}$ for every x. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* no x appears; the product rule applied as if both factors were functions of x earns 0.
2	$y = -\frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$, i.e. $y = -\frac{2}{\pi}x + 1$	1 pt $f\left(\frac{\pi}{2}\right) = 0$; 2 pts $f'(x) = \dfrac{-x\sin x - \cos x}{x^2}$; 1 pt $f'\left(\frac{\pi}{2}\right) = \dfrac{-\pi/2}{\pi^2/4} = -\frac{2}{\pi}$; 1 pt equation, any correct form. *Trap:* the point is on the x-axis, so the equation starts $y - 0$; reversing the quotient-rule numerator gives slope $+\frac{2}{\pi}$.
3	$x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$	1 pt $f'(x) = \sec^2 x - 4$; 2 pts $\cos^2 x = \frac{1}{4}$, $\cos x = \pm\frac{1}{2}$; 2 pts both solutions in the interval and nothing else. *Trap:* $\cos x = +\frac{1}{2}$ alone gives $\frac{\pi}{3}, \frac{5\pi}{3}$, neither in the interval; on this interval $\cos x < 0$, so only the negative root counts. Answering "none" earns at most 2.

Item	Answer	Rubric / trap
4(a)	5	Readings: $f(3) = 1$, $f'(3) = -2$, $g(3) = 2$, $g'(3) = 1$ (slopes are rise over run of the segment containing $x = 3$). $q'(3) = \dfrac{g'(3)f(3) - g(3)f'(3)}{[f(3)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* q has g on top: the f/g version gives $-\frac{5}{4}$; $g'(3)/f'(3) = -\frac{1}{2}$; reversed numerator gives -5; reading a height where a slope is needed.
4(b)	4	Readings: $f(-3) = -2$, $f'(-3) = 1$, $g(-3) = 2$, $g'(-3) = -1$ (slopes are rise over run of the segment containing $x = -3$). $p'(-3) = f'(-3)g(-3) + f(-3)g'(-3)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(-3)g'(-3) = -1$; $f(-3)g(-3) = -4$; mixing a value from one graph with a slope from the other.
5(a)	3	$h'(x) = f'(x)g(x) + f(x)g'(x)$, $5 = (-1)(4) + 3g'(2)$; substitute the given values and solve for $g'(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the product and asks for a factor's derivative; $h' = f'g'$ gives -5.
5(b)	5	$k'(x) = \dfrac{1 \cdot f(x) - x\,f'(x)}{[f(x)]^2}$, $-\frac{3}{4} = \dfrac{2 - f'(1)}{4}$; substitute the given values and solve for $f'(1)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* reversing the numerator gives -1.
6(a)	1	$h'(1) = \dfrac{f'(1)g(1) - f(1)g'(1)}{[g(1)]^2}$, with the row $x = 1$. 1 pt rule, 1 pt entries, 1 pt value. *Trap:* reversing the numerator gives -1; $f'(1)/g'(1) = -3$. Columns run x, $f'(x)$, $g(x)$, $g'(x)$, $f(x)$.
6(b)	0	$k(x) = f(1)\,g(3) = -10$ for every x, a constant. 1 pt, all or nothing. *Trap:* a product of two numbers; running the product rule with the $x = 3$ row earns 0.

Points: $10 + 5 + 5 + 8 + 8 + 4 = 40$.